**Lab 3**

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Engr 443 T2

**Documentation:** Used course notes and previous lab reports for formatting purposes. No unauthorized resources used. ChatGPT prompt can be found at

**Objective**

The objective of this lab is to design and evaluate a continuous time Kalman Filter to estimate a satellite’s pitch angle relative to the local horizon.

**Approach**

A generic FalconSat has been launched and stabilized using a gravity gradient boom, although there are still pitch oscillations that have not been removed yet. A horizon sensor on the satellite allows us to measure the pitch angle θ but the sensor is very noisy and doesn’t give individual measurements that are accurate enough for the control system on board. In order to determine accurate pitch angle states, a Kalman Filter is designed that incorporates those measurements real time and does a “best fit” to give a better estimate of the current pitch angle. To do this, the following steps are implemented.

1. A Simulink model to implement a Kalman Filter to estimate the pitch angle at each point in time is developed. To do this, an optimal observer is created using the gravity gradient state space equation.

2. A simulation using an initial value for Q of all zeros is run where the Kalman Filter output is compared to the raw measurements on the same plot. Additionally, the covariance matrix values are plotted.

3. A simulation using the Q matrix is run and the same plots in Task 2 are created.

4. A simulation using a unique “tuning” Q matrix is run to minimize uncertainty while maintaining accuracy is run, and the same plots in Task 2 are created.

**Assumptions**

The gravity gradient state space system is assumed to be uncoupled (i.e., the inputs do not directly affect the outputs without affecting the states). It is also assumed in our Simulink model that the satellite oscillates according to the gravity gradient pitch model given in equation 5. Finally, we assume that there are unmodeled perturbing forces in the real satellite from which the modeling and measurement errors are derived.

**Mathematical Techniques**

**Task 1:**

A continuous Kalman Filter that estimates satellite pitch angle is implemented in Simulink. The model is given in state space where:

(1)

(2)

where

– modeling errors

– measurement errors

Once the Kalman Filter is implemented, the optimal observer model becomes:

 (3)

 (4)

where





Next, the gravity gradient pitch equation is implemented into state space where the gravity gradient pitch equation is:

(5)

where

 - pitch angle

 - mean motion

 - moment of inertia about the roll axis

 - moment of inertia about the yaw axis

 - moment of inertia about the pitch axis

To implement this pitch equation in state space, the states are selected to be from which the following state space model is derived:

(6)

(7)

where

therefore,

Using the state space model in equation 6, a continuous time Kalman filter Simulink model is created to estimate satellite pitch states using the following constants from the FalconSAT satellite:

Moments of Inertia

*Ir* = 67.40 kg m2

*Ip* = 67.45 kg m2

*Iy* = 1.31 kg m2

Orbit Characteristics

alt = 560 km

inclination = 0º

eccentricity = 0

Horizon Sensor Data

Measurement standard deviation = 5º

The Simulink model is shown in Appendix A.

**Task 2:**

A simulation using the Simulink model developed in Task 1 and the following initial value of the Q matrix is run:

(7)

**Task 3:**

A simulation using the Simulink model developed in Task 1 and the following initial value of the Q matrix is run:

(8)

**Task 4:**

A simulation using the Simulink model developed in Task 1 and the following initial value of the Q matrix is run:

(9)

**Theoretical Predictions**

The continuous time Kalman filter is an algorithm used to estimate the state of a system evolving over time with linear dynamics and noisy measurements. It operates in two main stages: prediction and correction. During the prediction stage, the filter uses the system's dynamics to forecast the state and its uncertainty based on prior information. This involves propagating the state estimate forward in time and accounting for the system's inherent process noise, which reflects uncertainties in the model. This step utilizes the following equations:

|  |  |
| --- | --- |
|  |  |

where is the state transition matrix, is the state, and is the covariance matrix.

When a new measurement becomes available, the correction stage updates the state estimate and reduces its uncertainty. The filter calculates a quantity called the Kalman gain, which determines how much weight to assign to the new measurement relative to the prediction. It then uses this gain to correct the predicted state with the measurement, while also adjusting the associated uncertainty to reflect the improved estimate. The correction stage utilizes the following equations:

|  |  |  |
| --- | --- | --- |
|  |  |  |

where is a measure of how uncertain a model is prescribed to be. In the case of this lab, it represents the standard deviation of the horizon sensor of the satellite.

The Kalman filter operates continuously, integrating these two stages in real-time to provide an updated state estimate at every moment.

2. The description of how you determined the values for the matrices, A, C and W.

What we expect tasks to yield and why

**Experimental Results/Discussion**

**Conclusions and Recommendations**

Appendix A: Simulink Model

Appendix B: MATLAB Code